PREDICTING THE NATURAL FREQUENCIES OF REINFORCED CONCRETE SLABS USING ARTIFICIAL NEURAL NETWORKS

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ABSTRACT:

Floor vibration constitutes a critical serviceability concern in reinforced concrete buildings, necessitating accurate estimation of slab natural frequencies during the design stage. While analytical formulations yield reliable predictions, they are often computationally intensive and time-consuming, particularly for preliminary design and parametric analyses. This study develops an artificial neural network (ANN)–based model for the rapid prediction of natural frequencies of concrete slabs. A comprehensive dataset is generated from established analytical formulations by systematically varying key geometric and material parameters. The ANN is trained on normalized data and validated against independent test samples to assess its predictive performance. The results demonstrate that the proposed model accurately predicts the fundamental natural frequency, exhibiting excellent agreement with analytical solutions and achieving a coefficient of determination (R^2) exceeding 0.99. Owing to its high accuracy and computational efficiency, the proposed approach offers a practical and reliable tool for expedited vibration assessment of concrete floor systems.

Keywords: ANN, natural frequency, vibration, RC slab.

1. Introduction

Floor vibration has become an increasingly important serviceability concern in modern reinforced concrete buildings, particularly due to the widespread use of slender structural members, longer spans, and open-plan floor systems [1]. Excessive floor vibrations induced by human activities or mechanical equipment may not cause structural failure but can significantly affect occupant comfort and functionality [2]. Consequently, accurate prediction of the natural frequencies of concrete slabs is a critical requirement in the design and assessment of floor systems.

In recent years, data-driven approaches based on machine learning have attracted considerable attention in structural engineering applications. Among these approaches, artificial neural networks (ANNs) have demonstrated strong capability in approximating complex nonlinear relationships between input parameters and structural responses [3]. Owing to their ability to learn directly from data without explicit assumptions about the underlying physical model, ANNs have been successfully applied to various problems, including strength prediction, damage detection, and structural response estimation [4]. Nevertheless, the application of ANN techniques to the prediction of vibration characteristics of concrete slabs remains relatively limited in the existing literature.

Motivated by the need for efficient and reliable tools for vibration assessment, this study proposes an ANN-based framework for predicting the natural frequencies of reinforced concrete slabs. The proposed model is trained using a comprehensive dataset generated from finite element modal analyses, in which key geometric and material parameters are systematically varied [5]. By learning the implicit relationship between slab properties and natural frequencies, the ANN model aims to provide accurate predictions at a fraction of the computational cost required by conventional mathematical analyses.

The main contributions of this study can be summarized as follows: (i) development of a robust ANN model for predicting the fundamental natural frequency of concrete slabs; (ii) comprehensive evaluation of the ANN performance through comparison with mathematical results using statistical error measures; and (iii) demonstration of the practical applicability of the proposed approach for rapid vibration assessment in preliminary design. The proposed methodology offers a promising alternative to traditional numerical methods and can be readily extended to more complex floor systems and dynamic problems.

2. Theoretical basis

2.1. Vibration theory of concrete slabs

The dynamic behavior of reinforced concrete (RC) floor slabs can be idealized using classical thin plate theory under the assumptions of linear elasticity, small deflection, and homogeneous equivalent material properties. For serviceability-level vibration analysis, the reinforced concrete slab is commonly modeled as an equivalent isotropic plate with effective elastic properties.

1) Governing equation

For a thin rectangular plate, the free transverse vibration is governed by the Kirchhoff–Love plate equation [6]:

$$D\nabla^4 w(x,y,t) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = 0$$

Where:

- $w(x,y,t)$ is the transverse displacement;
- ρ is the mass density of the slab;
- h is the slab thickness;
- D is the slab thickness;
- ∇ is the biharmonic operator.

The flexural rigidity D is defined as:

$$\mathrm{D} = \frac{D_{rc} h^3}{12\,(1-\upsilon^2)}$$

Where:

- D_{rc} is the effective Young's modulus of reinforced concrete;
- υ is the Poisson's ratio of reinforced concrete, taken as 0.2.

The elastic modulus of reinforced concrete can be calculated from the elastic modulus of rebar and concrete using the following formula:

$$E_{rc} = E_b\left(1 + \mu\,\frac{E_s}{E_b}\right)$$

Where:

- E_b is the elastic modulus of concrete;
- E_s is the elastic modulus of rebar, acording to TCVN5574-2018 $E_s = 2 \times 10^5\,MPa$;
- μ is the rebar percentage.

2) Boundary conditions

For a slab simply supported along all four edges (SSSS), the boundary conditions are:

- $w - 0$ for all edges;
- $M_x = M_y = 0$ on the supported boundaries.

These conditions imply zero transverse displacement and zero bending moment along the slab edges.

3) Analytical Solution

Assuming harmonic motion:

$$w\,(x,y,t) = W\,(x,y)\sin\,(\omega,t)$$

and applying separation of variables, the mode shapes for a simply supported rectangular plate of dimensions $L \times B$ are:

$$W\,(x,y) = \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{B}\right)$$

Where $m,n = 1,2,3...$ is mode numbers.

Substitution into the governing equation yields the angular natural frequencies:

$$\omega_{mn} = \pi^2\sqrt{\frac{\mathrm{D}}{\rho h}}\left(\frac{m^2}{L^2} + \frac{n^2}{B^2}\right)$$

The corresponding natural frequencies are:

$$f_{mn} = \frac{\omega_{mn}}{2\pi}$$

For the first vibration mode:

$$f_{11} = \frac{\pi}{2}\sqrt{\frac{\mathrm{D}}{\rho h}}\left(\frac{1}{L^2} + \frac{1}{B^2}\right)$$

2.2. Artificial Neural Networks

2.2.1. Overview

Artificial Neural Networks (ANNs) are data-driven computational models inspired by the structure and functioning of biological neural systems. In structural engineering applications, ANNs are widely used to approximate nonlinear mappings between input parameters and structural responses without explicitly solving governing differential equations.

In the present study, ANN is employed to approximate the functional relationship:

$$f_{11} = F(L, B\,h, \upsilon, \rho)$$

Where is the first fundamental natural frequency of the reinforced concrete slab, and the remaining variables represent geometric and material properties.

2.2.2. Multilayer perceptron architecture

The most common ANN architecture for regression problems is the Multilayer Perceptron (MLP). An MLP consists of:

- An input layer;
- One or more hidden layers;
- An output layer.

Each neuron performs a weighted summation of inputs followed by a nonlinear activation function.

For a neuron j, the output is defined as [7]:

$$y_j = \phi\left(\sum_{i=1}^{n} w_{ij}x_i + b_j\right)$$

Where:

- x_i are the input variables;
- w_{ij} are the connection weights;
- b_i is the bias term;
- $\phi(x)$ is the activation function.

2.2.3. Training process

The ANN parameters (weights and biases) are optimized by minimizing a loss function. For regression problems, the most common loss function is the Mean Squared Error (MSE):

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - y_j^{ANN}\right)^2$$

The smaller the MSE, the better the model's predictive ability. In addition, the predictive ability of ANN models is also evaluated using indicators such as:

- Root Mean Square Error: $RMSE = \sqrt{MSE}$

Similar to MSE, the smaller the RMSE, the better.

- Coefficient of Determination (R-squared):

$$R^2 = 1 - \frac{\sum (y_i - y_j^{ANN})^2}{\sum (y_i - \bar{y})^2}$$

A value closer to 0 for R-squared indicates that the overall fit of the model is closer to reality.

3. Research model

3.1. Research objectives

This research aims to develop an approximate predictive model for the vibration problem of a single-supported reinforced concrete slab. This will help minimize computation time while still obtaining relatively accurate results.

3.2. Calculation tools

The ANN model was created using tools from Matlab software version R2016a. Additionally, the input data was calculated using numerical formulas in Excel.

3.3. Input data calculation parameters

The dataset was constructed from the following cases:

- $B(m) \times L(m)$: 1×4; 2×4; 3×4; 4×4 .
- $h(m)$: 0.08; 0.09; 0.1; 0.11; 0.12.
- $E_b\,(MPa)$: 27500 (B20), 30000 (B25).
- μ (%): 0.1, 0.2, 0.3, 0.4, 0.5.

3.4. Implementation process

- We built a dataset from Excel to train the ANN model, consisting of 200 samples.
- Conduct supervised training of the predictive model with a randomized 80% training - 10% validation - 10% testing ratio.
- Use the trained model to predict and compare the error.

4. Results and discussion

The ANN model used has the structure shown in Figure 1, consisting of two hidden layers and one output layer. The two hidden layers each contain 10 neurons, and the output layer contains 1 neuron. The two hidden layers utilize a hyperbolic tangent sigmoid activation function to help the model learn complex nonlinear relationships, while the output layer uses a linear function suitable for regression problems.

The results of training the ANN model are shown in Figure 2. The model converged after 873 iterations (epochs). This is a fairly large number, indicating that the network "persisted" in optimizing the weights to

Figure 1. Three-layer ANN model structure

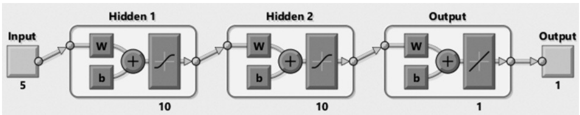
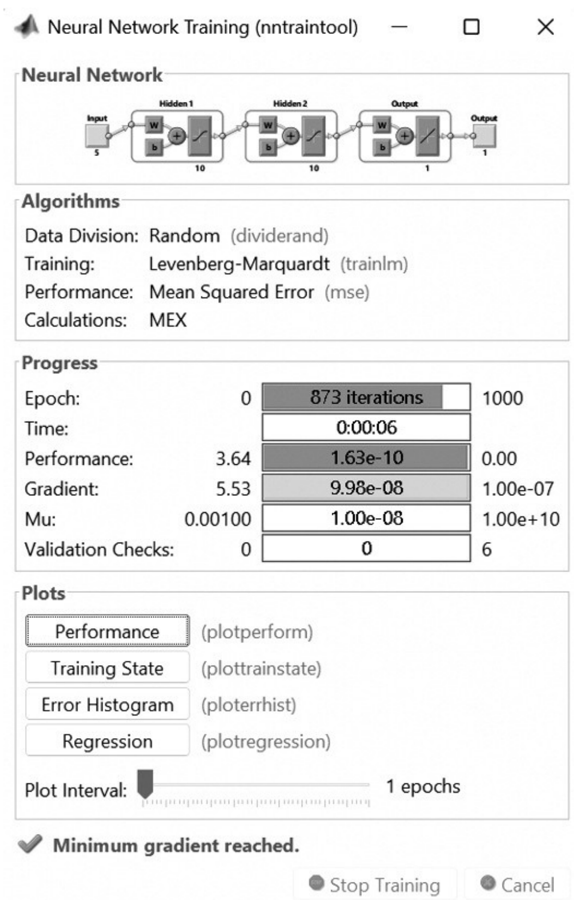


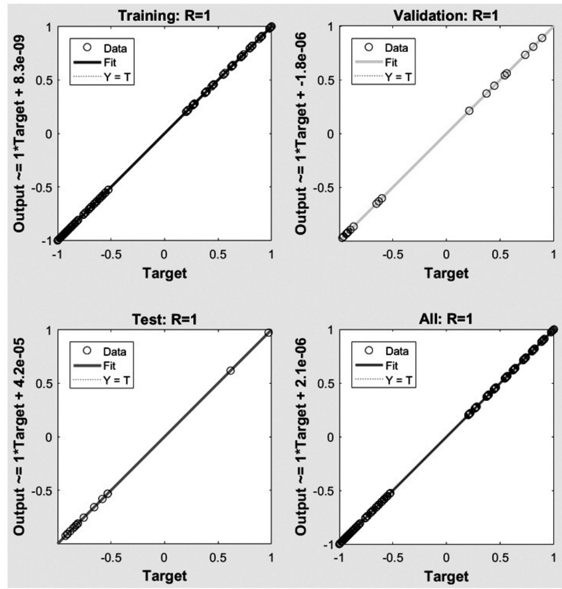
Figure 2. Training results of the problem after 873 rounds (EPOCH)



achieve the lowest possible error. The final MSE value reached 1.63e-10. This is an extremely small error level, showing that the model has almost become a perfect "digital twin" for the presented floor vibration data. The gradient reached 9.98e-10, meaning the model found the minimum point of the loss function. The Mu value shows that the network is converging very steadily towards the Newtonian optimal solution.

Figure 3 shows that all four graphs Training, Validation, Test, and All give an R value of 1. This confirms that the predicted (Output) and actual

Figure 3. The image illustrates a comparison between the results obtained from the ANN model and the actual (target) results of the problem in different datasets.



(Target) values match perfectly. The regression line (blue, green, and red lines) overlaps the ideal Y = T curve (dashed line). The model is ready for use and has the potential to replace complex simulation calculations to provide near-instantaneous oscillation frequency results with high accuracy.

In the next step, we used the trained ANN model to predict the natural vibration frequency of several cases. The comparison results are shown in Table 1. The comparison results show a very small error (maximum error of 0.016%), which indicates that the trained model performed effectively and can be used to quickly predict the natural vibration frequency of a single-supported reinforced concrete slab.

5. Conclusions

This study developed an Artificial Neural Network (ANN) model to predict the fundamental natural frequency of reinforced concrete slabs simply supported on four edges. The proposed ANN framework was constructed using five key input parameters representing the geometric and material properties of the slabs, enabling the model to capture the nonlinear relationship between structural characteristics and dynamic response.

Table 1. Predicting the natural frequency for several cases and compare the error

$B \times L \times h$	E_b	μ	f_{11}	f_{11}^{ANN}	Error (%)
$1\times4\times0.08$	27.5e3	0.1	130.943	130.943	0.000
$1\times4\times0.10$	27.5e3	0.2	164.269	164.271	-0.001
$1\times4\times0.12$	27.5e3	0.4	198.530	198.529	0.001
$2\times4\times0.08$	27.5e3	0.1	38.513	38.513	0.000
$2\times4\times0.10$	27.5e3	0.2	48.314	48.314	0.000
$2\times4\times0.12$	27.5e3	0.4	58.391	58.391	0.000
$4\times4\times0.08$	27.5e3	0.1	15.405	15.406	-0.006
$4\times4\times0.10$	27.5e3	0.2	19.326	19.323	0.016
$4\times4\times0.12$	27.5e3	0.4	23.357	23.357	0.000
$1\times4\times0.08$	30e3	0.1	136.724	136.727	-0.002
$1\times4\times0.10$	30e3	0.2	171.471	171.472	-0.001
$1\times4\times0.12$	30e3	0.4	207.114	207.113	0.000
$2\times4\times0.08$	30e3	0.1	40.213	40.213	0.000
$2\times4\times0.10$	30e3	0.2	50.433	50.432	0.002
$2\times4\times0.12$	30e3	0.4	60.916	60.915	0.002
$4\times4\times0.08$	30e3	0.1	16.085	16.086	-0.006
$4\times4\times0.10$	30e3	0.2	20.173	20.174	-0.005
$4\times4\times0.12$	30e3	0.4	24.366	24.366	0.000

A comprehensive performance evaluation was conducted by comparing ANN predictions with reference solutions obtained from mathematical formulations. The predictive capability of the model was quantified using statistical error metrics. The results demonstrated strong agreement between the ANN outputs and the analytical solutions, confirming that the trained network can accurately approximate the fundamental frequency within an acceptable error range. These findings validate the robustness, stability, and generalization capacity of the developed ANN model.

The proposed approach provides a computationally efficient and reliable alternative to conventional numerical techniques. Unlike traditional methods, which may require significant modeling effort and computational cost, the ANN model enables rapid prediction once trained. This characteristic makes it particularly suitable for parametric studies, preliminary design stages, and optimization tasks. Furthermore, the methodology can be readily extended to more complex slab systems, including plates with varying boundary conditions, heterogeneous materials, or additional dynamic effects such as damping and external excitation.

For future research, several directions are recommended. First, expanding the training dataset to include wider parameter ranges and different boundary conditions would improve model generalization. Second, hybrid approaches combining ANN with physics-informed constraints or optimization algorithms could enhance predictive reliability. Finally, integrating the developed ANN framework into structural health monitoring or vibration-based damage detection systems represents a promising application for practical engineering implementation.

Overall, the results confirm that ANN-based modeling is a viable and efficient tool for vibration analysis of reinforced concrete slabs and holds significant potential for broader applications in structural dynamics ■

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DỰ BÁO DAO ĐỘNG SÀN TỰ NHIÊN CỦA HỆ SÀN BÊ TÔNG THÔNG QUA MÔ HÌNH DỰA TRÊN MẠNG NƠ-RON NHÂN TẠO (ANN)

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TÓM TẮT:

Dao động sàn là một vấn đề quan trọng trong việc đánh giá khả năng sử dụng tại các công trình bê tông cốt thép, đòi hỏi việc ước tính chính xác tần số dao động riêng của bản sàn ngay từ giai đoạn thiết kế. Mặc dù các công thức giải tích cho kết quả đáng tin cậy, chúng thường đòi hỏi chi phí tính toán lớn và mất nhiều thời gian, đặc biệt trong giai đoạn thiết kế sơ bộ và các phân tích tham số. Nghiên cứu này phát triển một mô hình dựa trên mạng nơ-ron nhân tạo (ANN) nhằm dự đoán nhanh tần số dao động riêng của bản sàn bê tông. Bộ dữ liệu được xây dựng từ các công thức giải tích đã được thiết lập bằng cách thay đổi có hệ thống các tham số hình học và vật liệu chủ chốt. Mô hình ANN được huấn luyện trên dữ liệu đã chuẩn hóa và được kiểm định bằng các mẫu thử độc lập để đánh giá hiệu năng dự đoán. Kết quả cho thấy mô hình đề xuất dự đoán chính xác tần số dao động riêng cơ bản, có mức độ phù hợp rất cao với nghiệm giải tích và đạt hệ số xác định (R^2) vượt quá 0,99. Nhờ độ chính xác cao và hiệu quả tính toán vượt trội, phương pháp này cung cấp một công cụ thực tiễn và đáng tin cậy cho việc đánh giá nhanh dao động của hệ sàn bê tông.

Từ khóa: mạng nơ-ron nhân tạo (ANN), tần số tự nhiên, dao động, tấm bê tông cốt thép.